

INVESTMENTS & WEALTH

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MONITOR

THE SHAPES OF RETIREMENT PLANNING

Are You a Curve, a Triangle, or a Rectangle?

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The popular, professional, and academic literature on investing and retirement planning may confuse the casual as well as the dedicated reader because it reveals contradictory yet prescriptive findings.

The contradictory nature of the literature is especially troubling for advisors whose success is predicated upon offering informed, prudent, and defensible solutions to clients.

The purpose of this article is to sample the historical literature in order to (1) highlight contradictory prescriptions; (2) make observations, organized with geometric metaphors, that help resolve the apparent contradictions; and (3) suggest a new direction for ongoing research and professional practices.

Introduction

Our geometric metaphor tracks with academic history, which illustrates the following three-stage evolution—or, more precisely, co-development—in financial research:

- risk/return optimization: portfolio performance (curves)
- goals-based planning: sustainability (triangles)
- procedural prudence: feasibility (rectangles)

This progression in portfolio design approaches—from a curve, to a triangle, and then a rectangle—gives rise to the following questions:

1. Can an advisor find a practical and defensible method to help a client make prudent portfolio design and investment management decisions?
2. If yes, what is the shape of prudent financial advice?

It is our belief and experience that a client-focused discovery process creates a context in which the client can accommodate both quantitative portfolio analysis and qualitative, subjectively defined goals through methods that reconcile and transcend apparent contradictions.

Procedural prudence starts with the client and circles back to the client. The advisor helps the client organize facts and priorities in order to translate them into actions and outcomes. The fact patterns originate with client-specific data. The decisions are the client's to make, not the quant's or the psychologist's. Who makes the decision (governance) is as important as what is decided (policy) because a decision becomes successful, in large measure, only when it is willed into existence on a daily basis, by the client.

A careful reading of the research suggests the importance of several dimensions in prudent retirement investment planning, implementation, and monitoring. The proposed rank-ordering of these dimensions starting with first-things-first includes: (1) feasibility, (2) sustainability, and (3) portfolio performance monitoring and evaluation.

Here we assess each element's role in a successful retirement income practice.

Historical and Conceptual Review

The literature on wealth management over the past half-century roughly unfolds as follows:

- risk/return optimization (modern portfolio theory)
- goals-based planning (behavioral finance)
- procedural prudence (Household Balance SheetSM or HHBSSM)¹

Modern portfolio theory (MPT) draws on the systems operations tools and techniques developed during World War II (i.e., linear programming). Behavioral finance draws on a rich mine of psychological research about goals, motivations, and decision-making. Procedural prudence

draws upon observations in the assets/liabilities matching techniques for insurance companies, banks, and defined benefit plans and translates them into methods that develop the HHBSSM.

Each corresponds to a coherent point of view and an iconic graphical representation:

- the efficient frontier curve,
- the Maslowian triangle, and
- the HHBSSM rectangle.

A synthesis of the above observations yields the following:

- risk/return optimization: performance curves
- goals-based planning: sustainability triangles
- procedural prudence: feasibility rectangles

Intellectual history is, of course, not as tidy as this conceptual synthesis suggests. We merely use the metaphor of geometry to convey some complex ideas developed by people who have thought deeply on these subjects. Geometric metaphors are

a visual tool to organize and present the contradictory, and often contentious, points of view developed by various schools of thought.

The conceptual hierarchy used here is limited to the following three algorithmic steps:

- Input—what the practitioner needs to know about client goals, preferences, constraints, and other circumstances;
- Model—how the practitioner designs financial options suitable to the client; and,
- Output—how the practitioner presents financial choices so that the process serves the client’s best interests.

Input and output are discussed and debated often in the practitioner community. However, our choice of the term “model” over “theory” or “process” may require an explanation.

Knowledge may develop from metaphor, to model, to theory, and finally, to scalable process. Curve, triangle, and rectangle provide the metaphors. As for the concep-

tual hierarchy, we emphasize “model” rather than “theory” to mirror the current state of knowledge in the financial industry. Theory is a deep description of reality that predicts what will happen given a specific set of inputs. Models are incomplete descriptions of reality, and thus describe what should happen if the model were true (Derman 2012).

A fundamental limitation of financial research, due in part to the impossibility of creating reproducible experiments, is that we cannot really know, let alone prove, the fundamental theories. We cannot isolate the variables with sufficient predictive certainty to move decisively beyond the model stage (Taleb 2014).

Fortunately, what is consequential for the client is not theoretical truth; but rather, the impact of being exposed to or protected from risks. Given this perspective, we do not need to possess a perfect theory of risk in order to be effective. We can be effective with imperfect understanding because we seek to manage client exposures to the risks.

Examples of Debates (and Contradictory Prescriptions)	Are You a Curve? Modern Portfolio Theory (MPT)	Are You a Triangle? Behavioral Finance (BF)	Are You a Rectangle? The Household Balance SheetSM (HHBSSM)
Input [SIGNAL]	<p>What do you use as portfolio preference criteria? e.g., risk tolerance questionnaire, client utility, preference-free criteria</p> <p>How do you make use of investor utility? e.g., utility (concave/convex), marginal utility, risk aversion (absolute, relative), prudence</p>	<p>How do you use psychological aspects of risk and decision-making? e.g., prospect theory, mental account portfolio theory, etc.</p> <p>Do you use a behavioral portfolio theory utility curve? e.g., kinked (difference in slope) above/below a reference point</p>	<p>Do you use measures of FundednessSM? e.g., a first step before mapping (systematic and unsystematic) client risks, client constraints, and calculating the selected portfolio preference criteria</p>
Model [SYSTEM]	<p>Do you model stocks as safe (Siegel 1998) vs. unsafe (Bodie 1995) for the long-run? e.g., average vs. sum of random distributions, expected returns, variance, skew, kurtosis</p>	<p>Do you model at-risk only asset allocations; or, do you include insured (floored) solutions? e.g., conservative vs. dissipative distribution policy using annuities and immunized bond positions</p>	<p>Do you model (and monitor) risk capacity before short-fall probabilities? e.g., feasibility before sustainability</p>
Output [RESPONSE]	<p>What is the role of equity-weightings in asset allocation recommendations? e.g., buy-hold, constant-mix, constant proportion, declining or rising equity weightings</p>	<p>Do you recommend behavioral allocations? e.g., implementation as a single portfolio vs. implementation as multiple portfolios (e.g., goal-based asset allocation)</p>	<p>Do you recommend procedural allocations? e.g., mapping of risk exposures, risk management techniques allocations before asset allocations</p>

Further, the existence of contradictory prescriptions in the financial literature supports a “model” view rather than a “theory” view of financial knowledge. Table 1 presents prescriptive elements across two dimensions: schools of thought vs. algorithmic steps.

A detailed review of the selected examples of contradictory prescriptions shown in table 1, based on more than 50 peer-reviewed academic papers, is provided in the supplemental notes, beginning on page 5.

Table 1, together with the supplemental material, shows our focus moving from working from a mathematical basis; to integrating quantitative analysis more closely and transparently with client goals and preferences; and, finally, to integrating the process to reflect the client’s assets, liabilities, and risk exposures.

A client-focused approach develops methods to match prescriptive recommendations to specific client circumstances; or, in a behavioral framework, to specific client types.

Observations

Prudent decision-making is academically sound, administratively reasonable, and legally defensible. When we place the client, instead of the portfolio, at the center of the process, we resolve many of the prescriptive contradictions.

As we read academic and practitioner papers, keeping in mind the following question helps sort out the field in a way that eliminates most, if not all, of the contradictory prescriptions:

- Can you describe the ideal client for which the author’s specific prescriptive recommendation is appropriate given everything known about the client’s circumstances, goals, preferences, and constraints?²

This question prompts us to seek more complete client descriptions. As you read popular, professional, or academic papers

with this question in mind, you will see that some prescriptions seem to apply to stick-figure versions of clients, what novelists call “flat characters.” Other prescriptions seem to apply to more real types of clients, what novelists call “round characters.”

Flat vs. round client descriptions are the primary method to resolve prescriptive contradictions in the financial literature because they make it possible to better match specific prescriptions to specific client types. In this case, by client “type,” we do not mean “stereotype.” Rather, the term “type” encompasses psychological type (preferences), economic type (resource constraints), goal type (retirement savings, decumulation goals, etc.), and so forth.

Stick-figure characters can be readily pigeon-holed or stereotyped. On the other hand, round-figure characters think for themselves, react in ways that advisors may find surprising (especially when confronted with risk), and value well-grounded and on-point discussions concerning financial options.

Flat vs. round characters are a key feature of novels, TV, and movie scripts. Round characters are developed so that you can relate to them as real persons with a recognizable history, understandable goals, plausible motivations, internal contradictions, and the ability to change. The hero and the villain in a story are usually round characters. Think of Captain Kirk in *Star Trek*. Characters that make brief appearances are flat. They function as plot devices. They do not tell the full story. We only need to know a small number of dimensions of their personalities before they are removed from the narrative. Think of the Red Shirt characters in the original *Star Trek*, or Figwit in the *Lord of the Rings* movies. Often, the flat character client is a target for a single-product sales presentation; round characters, however, are better at making informed choices.³

Figure 1: Are You a Curve?

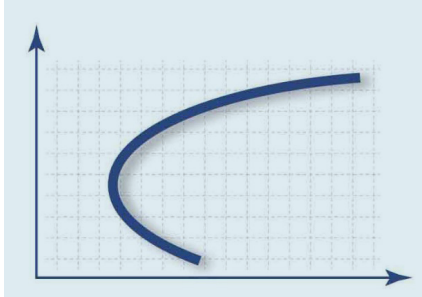


Figure 2: Are You a Triangle?

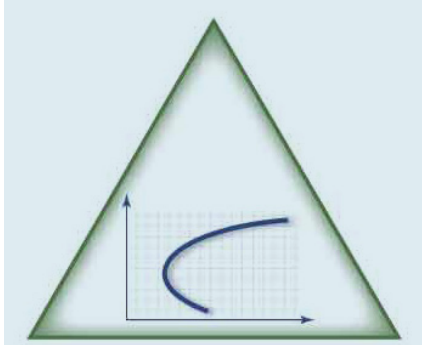


Figure 3: Are You a Rectangle?

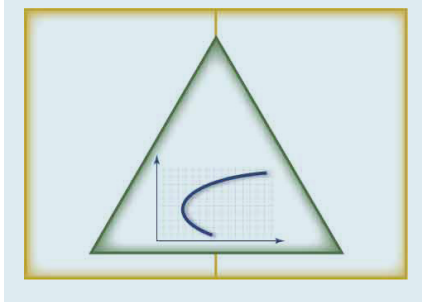
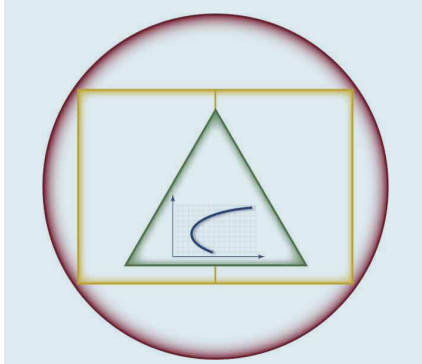


Figure 4: The Integration of the Curve, Triangle, and Rectangle Points of View in the Context of the Great Circle of the Client Advisory Process



As you can see from financial and literary examples, contradictory prescriptions become more manageable in the presence of round character descriptions. Round client descriptions help you help the client discover the ideal method for organizing and managing lifecycle wealth and liabilities. Descriptions that imply suitability for the average client, or that imply suitability for all clients as an undifferentiated mass, are considered flat descriptions because averages and generalizations are a measure of ignorance. Averages and generalizations are what one uses when the specifics of the client are not known. Round descriptions are about the specifics.

Forward Looking Statement

A comprehensive and integrative approach to retirement planning requires, according to common-law standards of prudence, the practitioner's exercise of reasonable care, skill, and caution as part of an overall investment strategy. Prudence is based upon a set of standards ranging from reasonable recommendations to justifiable processes provided in the best interest of the client.

Then we can ask, what is the shape of prudent financial advice? This shape includes the curve, the triangle, and the rectangle in the context of an iterative procedural prudence process focused on the client (see figures 1 through 4).

With this progression from curve to triangle to rectangle, advisors converge toward a round description of the client that encircles the scope of the advisory relationship.

Conclusion

The list of relevant dimensions for "roundness" in the development and monitoring of retirement plans includes the following dimensions:

- Feasibility (rectangle)
- Sustainability (triangle)
- (Portfolio) Performance measurement (curve)

The bottom two dimensions have received much attention from researchers in the past. The top dimension (feasibility) should receive more attention from researchers in the future.

What are the implications for financial advisors and consultants?

Helping clients find appropriate solution paths to their retirement needs and goals extends the process beyond investment portfolio design and implementation. The prudent advisor documents the following:

- Client goals that are feasible as opposed to those that fail the feasibility test
- Client goals that may be feasible currently but need to be sustainable throughout the requisite planning horizon
- Portfolio performance that encompasses both an evaluation of past performance and an assessment of the expected performance trajectory in order to provide adequate resources for contingent and/or unexpected expenses

This article is a call to advisors to expand beyond asset allocation pie charts and Sharpe ratio values, to diagnose feasibility of client goals, quantify the sustainability of goal funding, and monitor the plan's performance by helping clients ask and answer the following questions:

- Do I have enough to do what I'd like?
- How likely is it that my plan will remain sustainable under future economic environments?
- What is my capacity to meet the unknown or the unexpected? ●

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Endnotes

1. The household (or client or retiree) balance sheet is an economic balance sheet designed to value assets and liabilities that are both tangible and intangible at proper market values. RIIA has developed best practice templates for advisors, called the Household Balance SheetSM Views.
2. The development of this question was inspired by the work on business positioning developed by Flint McLaughlin (MECLABS). <https://meclabs.com/education/speakers>.
3. For an explanation of the Red Shirt character, see [https://en.wikipedia.org/wiki/Redshirt_\(character\)](https://en.wikipedia.org/wiki/Redshirt_(character)). For an explanation of the Figwit character, see: <https://en.wikipedia.org/wiki/Figwit>.

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NOTES TO TABLE 1: THE SHAPES OF RETIREMENT PLANNING

Collins and Gadenne (2017) draw from peer-reviewed papers covering a roughly 80-year time span and touch on important developments that have contributed to the art and science of prudent retirement planning. The notes that follow provide sources and comments for each element (i.e., cell) in table 1 of Collins and Gadenne (2017).

As noted in the article, readers can use table 1 to evaluate how their retirement planning process fits into a more general academic framework reflecting the curve, triangle, and rectangle organizational structure. These notes expand upon the questions posed in table 1, and they point toward a number of additional important issues that, explicitly or implicitly, influence the tenor of your retirement-advice process.

Table 1, Row 1: Notes about Input/Signal

Why would an investor, given the input provided to or gathered by the advisor, prefer one portfolio to another?

The input/signal row introduces, in the first cell, the idea of portfolio preference criteria. As you can see as you move across the row from left to right, curve-oriented

practitioners tend to use portfolio preference criteria that differ from those employed by triangle practitioners. The rectangle approach, in turn, integrates both approaches in an accounting-based framework that incorporates client contextual or client-centric aspects of behavioral finance (BF) with the quantitative approaches of modern portfolio theory (MPT).

Not surprisingly, the preference criteria used by the advisor (curve/triangle/rectangle) shapes the advisor's portfolio recommendations. Additionally, preference criteria determine advisors' preferred planning tools and communication techniques. For example, a preference criterion based on the amount of commission earned by a product sale may inform investment advice. The input/signal row spells out how advisors choose preference criteria, either explicitly or implicitly. The initial choices have meaningful consequences because they establish a decision-making framework that drives the answers (i.e., solutions) appearing in the output/response row: Which portfolio is best for the client given what we know about the client?

In this section, we do not advocate for any one approach; rather, we present and document, for the input/signal level of analysis,

some methods and approaches that an advisor can choose as a basis for the initial design of retirement portfolios.

Portfolio Preference Criteria

First, with an eye toward the ultimate financial goal, let's keep in mind that strategic asset allocation should provide an expected return equal to or greater than the return required for successfully funding the investor's long-term economic objectives. If an investor's threshold required return is 4 percent, it is probably imprudent, absent some unknown but compelling reason, to invest in an all-short-term Treasury-bill portfolio with an expected return of 1.5 percent.

This observation suggests that advisors should have, from the start of the portfolio building process, a clear understanding of their target performance measurement metrics. The metrics serve two functions: (1) to rule out recommendations that cannot lead to acceptable outcomes, and (2) to establish a reference point to evaluate the risk-return tradeoffs of remaining candidate investment solutions.

When considering a client's economic objectives in terms of the return required to

Table 1, Row 1: Input/Signal Row			
Examples of Debates (and Contradictory Prescriptions)	Are You a Curve? Modern Portfolio Theory (MPT)	Are You a Triangle? Behavioral Finance (BF)	Are You a Rectangle? The Household Balance Sheet SM (HHBS SM)
Input [SIGNAL]	<p>What do you use as portfolio preference criteria? e.g., risk tolerance questionnaire, client utility, preference-free criteria</p> <p>How do you make use of investor utility? e.g., utility (concave/convex), marginal utility, risk aversion (absolute, relative), prudence</p>	<p>How do you use psychological aspects of risk and decision-making? e.g., prospect theory, mental account portfolio theory, etc.</p> <p>Do you use a behavioral portfolio theory utility curve? e.g., kinked (difference in slope) above/below a reference point</p>	<p>Do you use measures of FundednessSM? e.g., a first step before mapping (systematic and unsystematic) client risks, client constraints, and calculating the selected portfolio preference criteria</p>

reach them, it is important to determine whether returns should be expressed either arithmetically or as multi-period compound returns. An advisor needs to document his or her point of view on this subject: Are you using compound (geometric) returns or arithmetic returns as the appropriate input/signal, portfolio preference metric?

In a multi-period setting, an approximation of compound return is:

$$\text{Compound Return} = \text{Arithmetic Return} - \frac{1}{2}(\text{Variance of Return})$$

This formula underlies the concept of “variance drain.” Return realizations differ from average (i.e., expected) returns in most circumstances. It is variance that makes you miss the average expectation; and, if you are not careful, you end up drowning while crossing a river with an average depth of two feet. Note the implication that, at a certain point on the risk-return spectrum, the investor may be able to create more long-term compound wealth by controlling risk (variance) than by reaching for additional expected return.

Compound portfolio returns equate to spendable cash. Compound returns are smaller than or equal to arithmetic returns; and the difference, when it exists, represents the impact of experiencing real-life, variable periodic returns. Neither investors nor advisors should take comfort in spreadsheet projections of average, constant returns during each period throughout the planning horizon.

As you look at the metrics for your various input variables, are they using consistent performance metrics? Is this consistent performance metric compound (geometric) return?

Second, selecting the initial portfolio preference criteria also sets the direction of subsequent investment recommendations. Clarity in the initial stages of investment analysis makes subsequent solution paths easier to find, compare, and assess. Quantifying portfolio return and risk parameters is important for determining suitable

strategic asset allocations (SAA). This creates a second decision point in the development of a retirement planning model:

The investor may prefer an SAA that is either the minimum risk allocation that meets the investor’s required rate of return or the SAA that maximizes investor utility (i.e., more money is better than less, but the risk of trying to maximize return may be too uncomfortable for the investor).

Trivially, owning the S&P 500 provides an investor with the expectation of generating more wealth than owning 50-percent aggregate U.S. bond market/50-percent S&P 500. More wealth is better than less wealth, but is it prudent for an investor to take more risk than that required to attain financial goals? Does the time horizon matter?

The utility-maximizing SAA on the mean-variance efficient frontier (a consequence of our first preferencing criterion) is the portfolio that maximizes investor utility:

$$U_m = E(R_m) - \frac{1}{2}R_A\sigma_m^2$$

Where:

- U_m = value of investor’s utility;
- R_m = expected return for asset mix m ;
- R_A = the investor’s risk aversion; and,
- σ_m^2 = the variance of asset mix m .

The formula for utility is close to the approximation formula for compound return—the difference is that a term is added for investor risk aversion. Investor welfare is positive in return and negative in volatility of return.

Compare the following:

- Compound Return = Arithmetic Return – $\frac{1}{2}$ (Variance of Return),
- with: $U_m = E(R_m) - \frac{1}{2}R_A\sigma_m^2$

The compound (geometric) return approximation (our first choice) is only using the first two terms (mean and variance) in a Taylor series expansion for estimating the value of a function. Other terms, the moments of a function, include skew, kurtosis,

etc. For bond positions, the first two terms in the matching Taylor series expansion are duration and convexity.

Advisors now face a third question: Are the returns of risky assets normally distributed? If the distribution of portfolio returns is not normal, then higher moments must be added to the series:

$$\text{Compound Wealth} = \text{Expected Return} - \frac{1}{2}\text{Variance} + \frac{1}{6}\text{Skew} - \frac{1}{24}\text{Kurtosis}$$

The point of these technical details is to suggest that the use of a Taylor series expansion in an MPT portfolio design context conforms precisely to an investor’s real-world objectives for compound wealth—or, for money to spend.

Behavioral finance observations correctly point out that the typical retirement income client is lost in less than 10 seconds whenever an advisor communicates such complicated portfolio construction and design principles in mathematical terms.

Given the above formula for calculating mean-variance utility (i.e., utility assuming a joint log-normal return distribution or utility for an investor with a quadratic utility of wealth function where risk aversion is the log of the investor’s marginal utility of wealth function), the portfolio with the highest utility value is preferred, all else being equal. Utility, under an MPT approach, becomes a critical preference criterion.

At which point the advisor faces a fourth choice, illustrated by the two questions that we selected for the first cell of table 1, row 1:

- What do you use as portfolio preference criteria? e.g., risk tolerance questionnaire, client utility, preference-free criteria
- How do you make use of investor utility? e.g., concave utility: relative vs. absolute risk aversion; state preference criteria, habit utility; Fisher utility², etc.)

According to MPT, every time an advisor makes a judgment regarding the

appropriateness of his or her financial recommendations—whether he or she realizes it or not—a utility of wealth function comes into play. Every time a client states a preference (“I want to make as much money as possible, as quickly as possible”), a utility of wealth function comes into play. The nature (domain and curvature) of the utility function, in turn, defines the client’s risk aversion, state preferences, certainty equivalent preferences, and so forth.

Some commonly used utility-based portfolio preference criteria reflect quantitative (relative or absolute risk aversion, state preference criteria, habit utility, Fisher utility), and/or preference-free criteria (stochastic dominance).

MPT also provides examples of portfolio preference criteria based on downside risk performance measures. A well-known example is Roy’s Safety-First Criterion.

Under Roy’s formula, the best portfolio is the one that maximizes the value of the safety-first ratio (the higher the ratio number, the better the portfolio):

$$\text{SF Ratio} = \frac{E(R_p) - R_L}{\sigma_p}$$

Where R_L = the lower bound acceptable return.

If one wishes to calculate a precise value for the safety-first ratio, one must assume a normal return distribution so that a standard Z-score probability can be determined.

However, utility is also important in a BF context. The research underlying prospect theory, for example, indicates that an investor’s decision-making process leads to utility functions rooted in psychology rather than strict rationality. This concept is often termed bounded rationality. An approximating MPT function might be the arc-tan function from trigonometry; the approximating BF function might be a function with one or more discontinuities—i.e., differing slope values directly above and below the reference point.

Behavioral Aspects

At which point, the advisor faces the choices reflected by the input/triangle questions in the second cell of table 1, row 1:

- How do you use psychological aspects of risk and decision-making? e.g., prospect theory, mental account portfolio theory, etc.
- Do you use a behavioral portfolio theory utility curve? e.g., kinked (difference in slope) above/below a reference point

During the 1970s, researchers extending the capital asset pricing model often reached conclusions that are largely compatible with those reached in BF-oriented studies. See, for example, Fishburn (1977).

This perhaps surprising observation reflects more than simply tweaking a rational model of investor decision-making to give it a bit more credibility. Rather, a careful reading of research studies indicates a gradual convergence of conclusions. This convergence, in turn, points the way toward a synthetic approach.

The synthesis explicitly and clearly emerges when Nobel laureate Harry Markowitz joined Das, Statman, and Scheid to argue that MPT and BF are, under many circumstances, fully compatible (Das et al. 2010).

This suggests, as an example of the progressive conceptual integration from the curve to the triangle to the rectangle, that it would be a mistake to think that the mathematical concept of investor utility is something only connected to the MPT school of thought.

Note: The concept of utility, as used in both MPT and BF contexts, encompasses a vast domain of research on and thinking about how people make decisions. Despite our emphasis on the movement toward a synthetic point of view, differences continue to exist

among the community of researchers into this subject. These differences are found both across and within various schools of thought.

A brief, and admittedly incomplete, list of outstanding issues includes the following:

- Are investor preferences ordinal (relative to a reference point, or to each other) or cardinal (capable of being mapped to a real number line), or both?
- Is MPT utility theory a descriptive theory (used primarily to model choice), a predictive theory (used to predict investor preferences), or a normative theory (used to overcome flaws in human decision-making)?
- What are the elements of “choice?” How are decisions made:
 - » when state-of-nature probabilities are known and payoffs (outcomes) are known—this is classical utility as first articulated by D. Bernoulli’s solution to the St. Petersburg Paradox (Bernoulli 1954);
 - » when investors are forced to select among several alternatives where, for each alternative, probability and payoff is well defined (roulette lotteries);
 - » when investors are unsure of probability but sure of payoffs (horse-race lotteries);
 - » when investors are unsure of both probabilities and payoffs (dependent on how information is presented)?
- Are decisions made by evaluating multi-dimensional alternatives; or, on an aggregate or a piecemeal basis (the portfolio perspective vs. evaluation in isolation)?
- How is risk defined relative to a reference point, an aspiration level, a target return level, a target wealth level, etc.?
- What is the role of an investor’s degree of confidence, subjective probability, decision weighting factors, etc.?
- How does one understand multi-stage (intertemporal) decision-making, for conflicting or competing objectives, under conditions of risk and uncertainty, with incomplete information regarding probabilities, consequences, or both?

Table N1: Portfolio Decision-Making and Preference Criteria	
Risk/Expected Return	<ul style="list-style-type: none"> Asset allocation policy—desired exposures to systematic risk factors Investment approach: active/passive/blend Asset management approach: asset-only/asset-liability management/behavioral portfolio theory/Household Balance SheetSM Currency risk management policy
Liquidity Needs	<ul style="list-style-type: none"> Spending policy: budgetary requirements Distribution policy: fixed \$ distribution / % of corpus distribution / constant \$ distribution / hybrid combination
Planning Horizon	<ul style="list-style-type: none"> Fixed or stochastic (e.g., longevity risk) Single stage or multistage—e.g., before and after paying off a mortgage. Asset dedication by objective (BF approach), or portfolio integration (MPT approach), or life-cycle approach Static SAA vs. dynamic SAA, or asset/liability [Fundedness] HHBS approach.
Tax Considerations	<ul style="list-style-type: none"> Tax management policy: tax loss harvesting/low turnover (index) vs. high turnover (active) management
Unique Needs, Preferences, and Circumstances	<ul style="list-style-type: none"> Investor risk aversion dictates asset management elections: buy & hold / constant mix / constant proportion portfolio insured rebalance policy Trade presentation and execution policy Limitations on asset allocation policy—permissible asset classes. Monitoring policy, manager selection, and retention policy
Legal and Regulatory Issues	<ul style="list-style-type: none"> Prescribes permissible and impermissible activities and tax strategies for many types of investors. Often constrains risk exposures, and often sets standards for prudent asset management. May dictate risk/return elections: e.g., total return vs. income and principal return.

One fact seems clear—both utility theory arising from the mathematical expressions (including game theory and decision analysis) of the rationalist school, and utility theory arising from the psychological explorations of the behavioral finance school (sociological and anthropological/evolutionary research) seem to exhibit poor out-of-sample predictive capability. Choices under conditions of risk and uncertainty seem to not be consistent, questionnaires and experiments appear to elicit differing results on different days even under “context-invariant” circumstances. Some are prompted to ask if utility and the risk-aversion curves that derive from it are merely theoretical, unobservable, and unprovable concepts. Perception of risk, and responses to it, may simply be investor reactions to changing information and op-

portunity sets. A unified theory continues to be elusive.

Third, the advisor now faces another input/signal task: Portfolio preference criteria influence the selection of an appropriate set of asset classes for an investor.

The BF approach often stresses this step in the portfolio design process. The gist of the BF approach lies in the observation that investors readily understand a portfolio construction process based on the principle that a safe asset can be matched to a critical objective. Whether a safe asset *should* be matched to a critical objective as the investor climbs a pyramid of goals has been a topic of intense debate and discussion.

This debate continues in the HHBSSM context in terms of “flooring.” Specifically, should flooring be actual or virtual; and what type of flooring best meets client needs?

Commonly used asset classes include:

- domestic common equity (large cap, mid cap, and small cap);
- domestic fixed income (intermediate term and long term)
- international common equity (developed and emerging markets)
- international fixed income (developed and emerging markets)
- alternative asset classes (real estate, private equity, natural resources, commodities, hedge funds)
- cash and cash equivalents

The above list may be restricted because of investor preferences and constraints (i.e., investment policy), because of legal concerns such as statutory requirements in state prudent investor laws, or because of specific provisions in irrevocable family trusts.

In the language of MPT (the curve), determining the extent to which the efficient frontier moves to the northwest—the improvement caused by increasing the opportunity set of investments—measures the impact of adding an asset class to a portfolio. That is to say, a useful addition to the opportunity set of investments should, all else being equal, allow the investor to achieve long-term financial objectives at more favorable risk/return tradeoff. This tradeoff is often measured by the improvement in the portfolio’s Sharpe ratio.

Remember that optimizing means optimizing to a specific normative hypothesis. MPT optimization techniques are based on several normative hypotheses (more choices for the advisor to make). Such normative optimization techniques include the following:

- mean-variance
- resampled efficient frontier
- Black-Litterman
- asset-liability surplus management
- simulation-based, etc.

Extensions include dynamic programming/optimal control approaches (often within a life-cycle context.)

In the language of BF (the triangle), asset selection policy is often a function of the priority a client assigns to goals. Critical goals are matched to low-variance assets; aspirational goals are matched to risky assets. The race is between the opportunity cost of heavily weighting low-risk/low-return assets and the return requirements necessary to sustain an adequate long-term standard of living.

All anchor and no sail is not necessarily a way to build a safe ship; all sail and no anchor is a recipe for disaster. This topic is revisited in the model/system row (row 2 of table 1). The accounting/financial statement context of HHBSSM helps clarify the nature of this debate, and helps advisors and clients focus on realistic and appropriate solutions.

Fourth, the list of portfolio preference criteria can be extended beyond portfolio performance metrics, specified utility functions, and other normative hypotheses to include the scope of diversification, tax considerations, regulatory demands—especially when an irrevocable family trust contributes to its beneficiary’s retirement income—shortfall risk metrics, liquidity needs, and so forth. This list forms the basis of the discussion found immediately below.

Tables N1 and N2 summarize relevant portfolio decision-making and portfolio preference criteria in an investment policy statement (IPS) or Retirement Policy StatementSM (RPS) context. You should be able to use the tables as a preliminary checklist to determine how well or poorly you have (1) understood client objectives, preferences, and economic circumstances; and, (2) how well your proposed portfolio addresses these criteria. If you flunk any of the categories, your recommended portfolio can probably stand some improvements. This statement holds true irrespective of whether you are a curve, a triangle, or a rectangle.

This decomposition of the IPS/RPS template indicates that selecting and justifying an appropriate strategic asset allocation

Table N2: Portfolio Decision-Making and Preference Criteria—Strategic Asset Allocation

Risk/Expected Return: For a retirement-income portfolio you are primarily concerned with: 1. feasibility 2. sustainability 3. flexibility (security)	<ul style="list-style-type: none"> Does the SAA generate an expected return equal to or in excess of that required to meet investor financial objectives? What are the relevant risk metrics: Standard deviation? Downside risk (dollar shortfall/percentage return)? Benchmark tracking risk? Liability benchmark risk? Fundedness?
Liquidity Needs	<ul style="list-style-type: none"> Does your SAA include nonliquid assets? What is the need for short-term liquidity? Is this need to be removed from the finite amount of investable assets prior to determining asset allocation?
Planning Horizon	<ul style="list-style-type: none"> Are alternative assets locked up beyond the planning stage timeline at which cash will be required?
Tax Considerations	<ul style="list-style-type: none"> Asset location vs. asset allocation decision. Should assets be selected via pre-tax or after-tax optimization?
Unique Needs, Preferences, and Circumstances	<ul style="list-style-type: none"> Is SAA macro-consistent? If not, what is the impact on rebalance policy/asset management elections? Should nontradable assets be considered (labor income/closely held business/personal residence)? Should SAA be age-related? What SAA maximizes investor utility—i.e., what are your portfolio preferring criteria?
Legal and Regulatory Issues	<ul style="list-style-type: none"> Do candidate asset classes meet legal and regulatory requirements?

(SAA) requires that you pay some attention to the interrelationships of the various sub-policies. Trivially, you could not readily justify a growth-oriented asset allocation based on long-term total return for a private trust if the trust document specifies that current beneficiaries are to receive accounting income and remainder beneficiaries are to receive principal.

Table N2 expands on table N1 with a focus on SAA:

A primary distinction between BF- and MPT-oriented approaches is in the selection process (types of assets and weighting priority given to each asset type within the overall portfolio) used to design, implement, monitor, and communicate financial information to clients. Irrespective of the input/signal school of thought (curve, triangle, or rectangle), the portfolio must have the expectation of meeting client objectives.

Fundedness

However, you can see that tables N1 and N2 still have an “accumulation” pedigree

and that retirement planning may require yet another level of integration between MPT and BF. This brings us, in particular, to the question in the third cell of table 1, row 1:

- Do you use Measures of FundednessSM? e.g., a first step before mapping (systematic and unsystematic) client risks, client constraints, and calculating the selected client preference criteria

The following discussion expands on the rectangle portion of the input/signal debates.

- The debate from the curve point of view focuses on definitions and mathematical expressions of utility curves that quantify the client’s expectations about reward and attitudes toward risk.
- The debate moves to the triangle point of view and focuses on the limits of utility formulas to describe the psychological aspects of client expectations and risk perceptions.
- Prefatory to MPT and BP formulations about utility, the rectangle view urges

consideration of client financial needs and resources.

One important dimension of developing “round” descriptions of a client is the “fundedness” measure. Three broad definitions are applicable to different household segments:

Annual consumption in retirement divided by total financial capital at retirement (C/FC): Defined as the ratio of a household’s (or individual’s) annual consumption needs to their financial capital or investable assets. A low C/FC indicates that the household (or individual) may be able to live on income from financial capital alone whereas a high value of the ratio means that the household (or individual) may have to rely on social capital or tap into non-liquid assets to meet the consumption needs.

Assets over liabilities (A/L): Utilizes the balance sheet as a tool to assess the adequacy of funding. The ratio of assets to liabilities compares the total assets including the financial capital as described above, social capital (value of one’s social circle), and human capital (value due to expertise, education, and skills), to consumption in retirement (includes expense needs, any debts, and other obligations). If assets exceed liabilities, the client is at least adequately funded. This particular measure of fundedness is also known as the funded ratio.

Salary replacement ratio (commonly known as the 80-percent rule of thumb): Suggests that if the annual estimated retirement income of a household (or individual) is equivalent to 80 percent of their final salary (last salary before retirement), the household (or individual) should be able to live their retirement comfortably.

Based on these definitions the households may fall into one of the following three segments defining the retirement-funding adequacy called fundedness:

Overfunded: An overfunded household can lead a comfortable retirement and is considered to be financially well prepared. These households can self-insure with diversified risk.

Constrained: A constrained household may have the risk of falling short on its financial needs during retirement. Such households need to protect a lifestyle floor with safe assets.

Underfunded. An underfunded household in most circumstances lacks the financial resources to keep up with the desired lifestyle. The underfunded household needs to budget for more savings while protecting a basic lifestyle through safe assets.

The three definitions for retirement readiness are applicable to different segments based on their fundedness and wealth levels.

- Wealthy households only need to focus on their financial assets and consumption in retirement, so they can use the C/FC measure.
- Less-wealthy households need to include all of their assets (including home equity) in the fundedness calculation and compare their assets to all claims against those assets in the future; therefore, they use a measure of fundedness called assets over liabilities (A/L).
- Finally, households with modest assets need rules of thumb on consumption and guidelines on budgeting in order to live within their means.

Measures of FundednessSM for retail advisory practices find their genealogy and foundation in the defined contribution business. Measures of fundedness entered into discussions within the defined contribution community as variants on the

definitions of the salary replacement ratio. Their purpose was to establish measures of retirement readiness as a justification for a specific level of savings during the accumulation phase of the client’s lifecycle.

The conceptual genealogy can be seen in papers such as Blake et al. (2013), which is of particular interest because it moves the debate from utility functions to targets and thresholds.

The internal debate in the defined contribution community is primarily focused on the percent (80 percent or other) that one should use.

However, the salary replacement ratio as a measure of fundedness has constraints that limit its use in the retail advisory process. As shown in PwC (2015), limiting constraints include the following:

- not capturing consumption patterns
- not capturing the balance sheet view
- offering a single point in time

The first generic category of measures of fundedness (C/FC) returns a yield-like measure that can be compared to the expected return on the portfolio. Its inverse, FC/C, returns an estimate of the potential longevity, in years, of the portfolio.

The second generic category of measures of fundedness is A/L. This includes measures such as NPV A – NPV L (Net Present Value of Assets – Net Present Value of Liabilities = Risk Capacity).

The third generic category of measures of fundedness is the salary replacement ratio.

Each measure exhibits strengths and weaknesses whose relevance can be matched to specific types of clients. Advisors can use all measures of fundedness; but, best practice focuses retirement analysis with a measure of fundedness that is most appropriate

“The three definitions for retirement readiness are applicable to different segments based on their fundedness and wealth levels.”

for a specific type of client (PwC 2015; Rao et al. 2015).

For instance, reliance on C/FC would be appropriate for high-net-worth/overfunded clients. Reliance on A/L would be appropriate for affluent and constrained clients, and salary replacement would be appropriate for mass-market and underfunded clients.

Measures of fundedness are an important first step in the creation of a “round” client profile whose initial overfunded/constrained/underfunded status informs the selection of questions that follow, including the choice of rational and psychological utilities.

In particular, the A/L measure of fundedness provides an introduction to the related topic of risk capacity. Both A/L and risk capacity are related to the household balance sheet. A/L is a ratio of assets to liabilities. Risk capacity is a subtraction of the net present values of liabilities from the net present value of assets. We will present a fuller discussion of risk capacity as we discuss the model/system row of table 1.

Table 1, Row 2: Notes about Model/System

In this section, we touch on several issues of importance to retired investors, and on techniques and approaches available to those who advise them. The techniques and approaches reflect a variety of assumptions underlying the model/system—either implicit assumptions regarding the return evolutions of financial assets and, in some contexts, economic liabilities; or, explicit use of life-cycle or simulation-based models that also incorporate a veritable host of assump-

tions. The heterogeneity in approaches and opinions makes it clear that there is no one-size-fits-all solution to meeting the retirement income challenge. Given space limitations, we can only discuss a few aspects of a much larger debate.

The optimal weighting of equity in a retirement portfolio is a particularly contentious—and complex—topic. Most advisors are familiar with conventional wisdom (rule of thumb) nostrums such as: the percentage of equity exposure should equal 100 minus the investor’s current age. The academic literature reveals a propensity to recommend, especially in the pre-2001 literature, a high percentage of equity in a retirement portfolio.

The recommendation flows from several assertions, based on both theoretical and empirical considerations, including: (1) stocks protect purchasing power because they offer inflation-beating returns; and (2) a high weighting to equity within a portfolio’s asset allocation effectively mitigates shortfall risk (e.g., probability of depletion during the investor’s life span) and can support higher standard-of-living goals.

At least two counterpoints exist:

1. An argument championed by Paul Samuelson, Zvi Bodie, and other stalwarts of the MPT school of thought. They argue that time does not reduce the risk of stocks; indeed, the long run is merely an aggregation of short runs, which, in essence, means more opportunities for things to go wrong.
2. An argument championed by the BF school of thought advocates for

behavioral portfolios based either on multiple goal segregation (goals-based investing) or on matching critical objectives with low variance investments (a pyramid-like structure). They argue that, for investors lacking a suitable background in financial economics, MPT-based portfolios, often with larger equity allocations, seem to exhibit both an opaque structure and a disconcerting level of volatility. Such a combination pushes even the most-determined investor toward abandoning the portfolio during economic downturns.

The advisor has to make yet another choice: What role do stocks have within a portfolio where an important investor objective is the long-term sustainability of a living standard in retirement?

Ultimately, the source of income is principal—to what extent does a decline in portfolio value during a bear market translate directly into a decline in an investor’s ability to sustain target income? As a general rule, the importance of maintaining purchasing power looms large for retired investors. Individuals have minimum standard of living requirements that must be funded throughout a potentially long period of retirement life (longevity risk). Future standard of living depends on the ability to purchase the required goods and services, and failure to maintain a minimum standard of living produces negative utility for retirees. Therefore, it is the purchasing power of wealth—as opposed to the nominal dollar amount of wealth—that is a critical variable in retirement income planning.

Table 1, Row 2: Model/System Row			
Examples of Debates (and Contradictory Prescriptions)	Are You a Curve? Modern Portfolio Theory (MPT)	Are You a Triangle? Behavioral Finance (BF)	Are You a Rectangle? The Household Balance Sheet SM (HHBS SM)
Model [SYSTEM]	Do you model stocks as safe (Siegel 1998) vs. unsafe (Bodie 1995) for the long-run? e.g., average vs. sum of random distributions, expected returns, variance, skew, kurtosis	Do you model at-risk only asset allocations; or, do you include insured (floored) solutions? e.g., conservative vs. dissipative distribution policy using annuities and immunized bond positions	Do you model (and monitor) risk capacity before shortfall probabilities? e.g., feasibility before sustainability

For example, for long-term investors, some commentators suggest that stock returns offset the loss of purchasing power caused by inflation (Gastineau et al. 2007). This is in contrast to bonds, which, according to Gastineau et al., exhibit returns “negatively related to inflation.”

Thus, although retirees prefer to keep their nest eggs safe from loss of principal, there may be compelling reasons to diversify beyond cash and bonds:

1. Safety of principal is not the same as preservation of the principal’s purchasing power.
2. The long-run returns from stocks are more likely to outpace inflation than the long-term returns from cash or short-maturity bonds.

These arguments mirror those found Jeremy Siegel’s popular book *Stocks for the Long Run* (1998):

It is widely known that stock returns, on average, exceed bonds in the long run. But it is little known that in the long run, the risks in stocks are less than those found in bonds or even bills! ... Real stock returns are substantially more volatile than the returns of bonds and bills over short-term periods. But as the horizon increases, the range of stock returns narrows far more quickly than for fixed-income assets ... Stocks, in contrast to bonds or bills, have never offered investors a negative real holding period return yield over 20 years or more. Although it might appear riskier to hold stocks than bonds, precisely the opposite is true: the safest long-term investment has clearly been stocks, not bonds.

Although stock returns may, or may not, offset inflation, the role of stocks—and hence their optimal weighting in a portfolio—is a function of whether the investor implements a strategy:

- to maximize risk-adjusted expected return, or
- to optimize investment surplus subject to the constraint that fundedness should remain positive, or

- to establish a residual pool of risky assets after a portfolio’s fixed income allocation funds critical objectives.

To the extent that a portfolio holds equity investments, the investor anticipates that he will, on average, earn a return higher than that available from fixed income investments. Therefore, over the applicable horizon, funding with equity investments should generate more spendable cash than an equivalent-value portfolio funded with bonds. However, equity investments have more volatility (and a different degree of interest-rate sensitivity) than fixed income investments. At any particular point in time there is a greater risk that the value of an equity-loaded portfolio will be less than the present value of the retired investor’s projected liabilities. A heavy allocation to stocks may drive a retired investor into technical insolvency.

Thus, the advisor faces a choice, as suggested by the first cell of the model/system row:

- Do you model stocks as safe (Siegel 1998) vs. unsafe (Bodie 1995) for the long-run? e.g., mean reversionary process vs. random walk process vs. sub-Martingale process

The role of risky stocks within the financial asset portfolio takes on new dimensions within an asset/liability management (ALM) context. ALM entails a different approach to portfolio design and asset management. A portfolio represents a finite amount of capital that must be managed carefully so that it successfully discharges a series of liability payments. The ALM approach is perhaps easier to understand within the context of a corporate defined benefit (DB) pension plan. We outline three variations on this theme:

- a financial engineering approach (Zvi Bodie)
- a surplus optimization approach (William Sharpe)
- an actuarial (risk-factor) decomposition approach (Aaron Meder and Renato Staub)

Although each approach finds its origins in investment planning for institutional investors, each has a direct extension to individual wealth management.

Financial Engineering

In the well-known textbook *Investments* (Bodie et al. 2008), the authors suggest that bonds may be preferred over equity because of the ability to immunize liabilities through duration/convexity management strategies. If the cash-flow obligations are not immunized, the implied costs (the call on corporate assets to make up funding shortages) act as a “self-insurance” liability, the value of which may equal or exceed the expected excess return offered by equity investing. It is as if the plan owns a put on corporate assets.

Bodie et al. (2008) propose that only equity investments positioned above the capital market line (CML)—as defined by Sharpe—add value, but this strategy involves active security selection and market timing skills.

In contrast with active management and market timing, merely matching market returns (indexed investing) neither mitigates shortfall risk nor decreases plan- unding costs (despite the fact that equities have a higher expected nominal return), because equities must carry a higher discount rate (“k” in the Gordon dividend discount model: $p = d/(k - g)$) to reflect their higher risk. Thus, although there may be valid reasons for funding a DB plan with a high allocation to equities, it is imprudent to implement equity asset allocation targets based, in the Bodie et al. (2008) authors’ opinion, on the following *incorrect* assumptions:

- Equities are not risky in the long run.
- Equities are an adequate inflation hedge. (Counter-argument: “A U.S. investor in the S&P 500 in 1967 did not recover purchasing power until about 13 years later. The loss of purchasing power during that period, peak to trough, was about 25 percent.”)

Surplus Optimization

Sharpe and Tint (1990) introduced the term “liability hedging credit”; a portfolio with a

positive credit provides a utility benefit “... exactly analogous to, but in the opposite direction from, a risk penalty.” The credit motivates plan sponsors to form optimum portfolios by taking advantage of covariance between the assets and liabilities. The magnitude of the credit’s benefit is a function of the investor’s risk tolerance and the ratio of current assets to current liabilities. It leads to a process of portfolio surplus optimization. Using a standard mathematical expression for expressing portfolio optimization, the ALM approach becomes:

$$\text{Maximize Expected } (R_A) - \frac{\text{Variance}(R_A)}{t} + 2 \frac{kL_0}{tA_0} \text{Covariance } (R_A, R_L)$$

Or,

$$\text{Utility} = \text{Expected Return} - \text{Risk Penalty} + \text{Liability Hedging Credit}$$

where the risk penalty is variance divided by the investor’s risk tolerance t , and k represents an “importance” factor ($1 =$ preference for full surplus optimization). All else equal, asset returns exhibiting high correlation with liability price change provide a hedging—i.e., risk reduction to surplus—benefit.

The ALM approach demands an asset management strategy that explicitly models liabilities.

It is important to re-read the sentence above because this point is critically important for the development of *round* descriptions of the client as a means to resolve the contradictory prescriptions in the financial literature.

A plan sponsor seeks the optimal asset allocation where “optimal” is defined relative to funding the liabilities. The approach considers both the magnitude of expected return and the relationship of changes in asset values relative to changes in liability values. Sharpe and Tint (1990) motivated further research in the area of ALM for institutional portfolios owned by corporations, banks, and insurance companies. For DB plans, ALM focuses on the surplus efficient frontier where surplus is defined as

the difference between the current market value of assets and the present value of projected liabilities.

Although approaches akin to the one outlined by Sharpe and Tint (1990) follow a Markowitz mean-variance optimization format, a major difference is that the objective function is to minimize the variability of the plan’s surplus subject to constraints on funding and/or minimum surplus-to-asset ratios—the funding ratio. In this context, the portfolio that minimizes

surplus variance is termed the minimum-surplus variance (MSV) portfolio. In terms of a DB pension plan, this portfolio is a fixed income portfolio that consists either of a cash-flow matching bond sequence, or an immunized bond portfolio wherein the duration of the fixed income assets matches the liability duration. For a plan with a long-duration liability structure, the preferred (and safe) asset is not a Treasury bill, but rather a long-term bond. The liability acts like a short position in long-term bonds, whereas the corresponding asset portfolio acts like a matching long position. The Sharpe ratio, as a portfolio preference criterion, may no longer be an allowable metric.

If the discussion seems, thus far, to ignore the tenets of behavioral finance, it is largely because it is rare to find an institutionally owned portfolio constructed according to BF principles. The exception is discussions about corporate-sponsored, participant-directed, defined contribution (401(k)) plans such as Blake et al. (2013).

The world of ALM brings us squarely back to the proposition that prudence cannot be defined according to labels (safe investments). Evaluated in isolation, most investors would not characterize long-term bonds as “safe” assets. Rather, the prudent asset manager seeks to develop strategies that enhance the probability that a finite

amount of capital will be sufficient to discharge the client’s objectives.

Actuarial Risk-Factor Decomposition

Meder and Staub (2007) decompose a DB pension plan into its actuarial components. This is a helpful way to see how and why DB plans require actuarial certification regarding their funded status and their contribution/funding requirements. The authors assert that “asset-only” (AO) optimization neglects the liability side of the balance sheet. An AO approach implicitly assumes that plan liabilities have no market-related risk exposures. However, it is well-known that liability valuation is a function of the applicable discount rate, which, in turn, is a function of interest rate and inflationary forces that impact most tradable financial assets.

Meder and Staub (2007) list relevant risk factors. From the asset-only perspective, the nominal dollar safe asset is cash or short-term Treasuries. From the liability-relative perspective, the safe asset is a liability-mimicking security. From a short-term perspective, a plan sponsor defines risk as the possibility that the mimicking portfolio will fail to hedge liability price changes over the forthcoming period. A short-term hedging perspective suggests that the ideal asset portfolio consists of duration-matched bonds so that the change in values are offsetting. However, from a long-term perspective, the variability in the real dollar value of benefit payments is also a vital concern. Benefits are likely to increase and, therefore, the asset portfolio should account for inflation and growth factors in compensation—which, in turn, drive growth in dollar benefits. A good portfolio design model must deal with both planning horizons.

The authors provide a list of DB plan costs. They note that costs for inactive participants (primarily retirees) usually are not indexed for inflation and, therefore, constitute a fixed liability that usually can be hedged by a nominal bond portfolio. If, however, post-retirement benefits are indexed, the hedging asset portfolio for this

plan expense is an inflation-linked bond portfolio (e.g., Treasury inflation-protected securities or TIPS).

For active participants, plan costs divide into payments attributable to past service credits (accrued benefits) and payments for projected future service credits (future compensation projections including wage growth and wage inflation). Accrued benefits are already “in the can” and, therefore, unless indexed to inflation, can be hedged by a portfolio of nominal bonds. By contrast, the future wage liability is linked to the FAS 87 Projected Benefit Obligation calculation. Most DB plans calculate final benefit payments based on service credits that increase over time (e.g., are not strictly linear with respect to time of service), and that apply to a higher future wage base. Plan sponsors can hedge future benefit costs using a combination of nominal bonds, inflation-adjusted bonds, and equities.

The gist of the authors’ recommendations is to establish a matrix multiplication structure in which the risk factor adjustment vectors pre-multiply a variance/covariance matrix. The values of the matrix derive from proxies taken from the real economy; and it uses the proxy’s time series of returns to represent the financial risk factor to which the plan is exposed (changes in real rates, inflation, wage growth, equity risk premiums, and real and nominal bond risk premiums). The Meder and Staub (2007) methodology parallels that often used to create a linear multifactor model (e.g., an arbitrage pricing theory model) in which the model builder identifies macroeconomic explanatory factors driving financial asset price changes. The next step determines the sensitivity of plan assets and liabilities to each of the factors—i.e., determines the value of factor betas. Residual risks (noise) are assumed to be uncorrelated with all other terms in the matrix. The theoretically optimal asset allo-

cation is the mimicking asset portfolio that best tracks changes in liability values.

Given the decomposition of cost factors for DB plans, it is readily apparent that the optimal mimicking portfolio will hold primarily a lower expected return position in bonds as opposed to a higher expected return position in equities. As a practical reality, in the authors’ opinions, such a portfolio will prove unacceptable to many plan sponsors because of the relatively higher costs required to fund benefits—i.e., maintain a positive funding ratio.

To recap, the Bodie et al. (2008) approach may best be seen as a financial engineering approach where the authors define a self-insurance option and then use valua-

“The three approaches provide a rich level of insight into both investment theory and practice. There may be no universally right, or even best approach to prudent asset management. Nevertheless, the prudent advisor should be capable of demonstrating why his or her management strategies are legally defensible, academically sound, and administratively reasonable.”

tion theory to quantify the option’s costs and risks; the Sharpe and Tint approach is an extension of the classic Markowitz optimization algorithm for solving the portfolio selection problem (optimizing surplus); and the Meder and Staub approach is an actuarial approach (decomposition of a defined benefit plan’s cost factors) and requires multifactor risk model building.

The three approaches provide a rich level of insight into both investment theory and practice. There may be no universally right, or even best approach to prudent asset management. Nevertheless, the prudent advisor should be capable of demonstrating why his or her management strategies are legally defensible, academically sound, and administratively reasonable.

As stated, the three institutional portfolio management approaches find counterparts in private wealth management:

- a financial engineering approach
- a surplus optimization approach
- an actuarial (risk-factor) decomposition approach

Financial Engineering Approach

For the financial engineering approach as applied to personal retirement income planning, Bodie (1995) counters the stocks-are-the-safe-asset for a long-term investor argument by using option pricing theory. Specifically, he evaluates the riskiness of maintaining a position in risky assets in terms of the cost of a put option insuring a return in excess of the risk-free

rate. The cost of a put option guaranteeing a target rate of return rises over time irrespective of whether stock price changes are a random walk or a mean-reverting process: “If stocks were truly less risky in the long run than in the short run, then the cost of insuring against earning less than the risk-free rate of interest

should decline as the investment horizon lengthens. But the opposite is true.” Rather than blindly following the conventional wisdom that young investors should hold a high percentage of their portfolio in equity, Bodie argues that such asset allocation advice may not apply if (1) labor income is highly correlated to equity risk; or (2) if the investor is in danger of falling below “... some minimum subsistence level of consumption. People should be expected to insure against falling below such a level through their asset allocation policy.” The Bodie (1995) argument is an important underpinning for advisors opting to initiate a “flooring” approach to retirement financial planning.

As a side note, we acknowledge the controversy surrounding the concept of flooring —

some practitioners arguing for an establish-the-floor-at-the-first-opportunity approach; others, at the other end of the spectrum, asserting the wisdom of maintaining a higher expected return portfolio of risky assets as long as portfolio value is on the plus side of the virtual floor. The first argument relies on a safety-first approach to portfolio management; the second relies on an opportunity cost argument. But this is exactly the issue acknowledged by Meder and Staub—the “optimal mimicking portfolio will hold primarily a lower expected return position in bonds ...,” but such a portfolio will prove unacceptable to many plan sponsors because of the relatively higher costs required to fund benefits—i.e., maintain a positive funding ratio.

We also note that many BF commentators assert that the rationalist approach developed for institutional investors does not translate well into the world of private wealth management, because the individual investor’s economic circumstances are more complex and variable. We suspect that most corporate financial departments would be surprised at such a statement given the multitude of stakeholders in corporate decision-making, multiple and conflicting objectives within various factions of the stakeholder group, and the myriad complexities—both economic and political—to evaluate.

This essay is not the place to debate the merits of various flooring strategies. Zwecher (2010) provides a good review of the many issues involved in flooring strategies.

However, one interesting financial engineering approach—perhaps applicable to private wealth management—is found in Ang et al. (2013). This article, discussing optimization of pension plan surplus, presents an option valuation approach to the following question: Given (1) a DB plan’s current funded status [assets ÷ liabilities], and (2) a plan’s asset allocation, how much would the plan sponsor pay for a put option the payoff of which is the plan’s future shortfall amount, if any?

[Payoff equals (Maximum of end-of-period Liabilities) – (end-of-period Assets), 0)]

This question can be generalized to consider the value of a shortfall put option for an individual’s retirement income plan where the value of the lifetime cash-flow liability is measured by an actuarial (annuity cost) benchmark. Given the current funded status of an individual’s retirement plan, what is the current market value of a put option calibrated to eliminate future shortfall risk?

The option valuation approach provides an interesting way to compare and contrast a variety of planning methodologies including behaviorist-oriented flooring recommendations, e.g., a bucketing/laddering strategy via bonds, or annuitizing to secure a threshold periodic income.

What is the intrinsic value of a protective put, where the strike price is based on a stochastic lower-bound terminal wealth level? What is the opportunity cost of locking in a permanent budget constraint via a flooring strategy? Although such a put contract does not exist in the marketplace, calculating fair value given its provisions should provide great insight into the costs for and reasonableness of protection/flooring strategies. We invite graduate finance students to have a go at it.

Surplus Management Approach

The surplus management approach is more commonly followed by practitioners from the rationalist school using the tools and techniques of MPT—especially, optimal control theory and its closely related cousin dynamic programming—to define, model, and manage an individual investor’s consumption surplus (assets – liabilities). In an interesting extension of ALM, practitioners from the behavioral finance school, using the tools and techniques of goals-based investing, advocate for a variety of asset-matching/immunizing techniques including flooring, bucketing, and dedicated portfolio strategies.

One rationale underlying both the MPT and BF schools lies in the realization that the

central limit theorem (CLT) does not guarantee long-term success. The CLT implies that as you add up more and more independent draws from a population (e.g., a collection of random variables), the distribution of results approaches a normal distribution. Unfortunately, the CLT provides comfort to the investor only in the case where the investor can average the long-term results of many long-term periods. Investors, however, are stuck with only one result that unfolds over a single long-term period.

These distinctions are important for understanding the debate over the question: Does time reduce risk? Consider flipping a fair coin (i.e., a proxy for an independent return series). If you flip it five times, you may hit all tails (with tails signifying a monetary loss). If you flip it many times (diversification over time) do you reduce your risk? Certainly, the law of large numbers suggests that as many coin flippers approach an infinite number of flips, the aggregate proportion of heads to tails approaches 50/50. We can predict that the average investor will come close to breaking even. However, at the limit, the chances of any single investor breaking even by flipping an infinite amount of times approaches zero. Stated otherwise, rate of return converges to the mean, levels of wealth diverge away from the mean.

If you flip a coin 1,000 times, you break even only with exactly 500 heads and exactly 500 tails, which, given the number of flips, is an extremely improbable result. Despite the fact that coin toss participants have a mathematical expectation that they will leave the game with zero profit, the possibility of leaving the game with wealth significantly different than zero *grows* with time (i.e., with the number of tosses or Bernoulli trials). The distribution of final results for this game, having a mean of \$0 and a variance of \$1, is a normal or bell curve distribution given a sufficiently large number of trials (the binomial or Bernoulli distribution approximates, at the limit, the bell curve distribution).

The variance term, however, will push actual results for any single player away

from the mean of the distribution, which is \$0.00. If the variance term pushes the player toward the left-hand side of the bell curve, the player is in negative (loss) territory; if it pushes the player to the right-hand side, he or she is in positive (profit) territory. If the game is limited to a few tosses, many players will break even. However, after a sufficiently large number of tosses, it becomes virtually unthinkable that any player will break even (expected value is a value never to be expected). Thus, for a game of 100 tosses, there is a 5-percent probability that the player will either win or lose \$20 $[(\$1 \text{ variance}) \times (2 \text{ standard deviations on the bell curve}) \times (100)^{1/2}]$; for a game of 1 million trials, there is a 5-percent probability that the player will either win or lose \$2,000. The longer you play the game, the greater the odds that you will be far away from the expected value.

“Asset management strategies that seem appropriate in the short term may undercut a portfolio’s chances of succeeding in the long term.”

It seems that time, on average, decreases risk over the population average; but, for any given individual, time increases risk. This stylized example may temper the inclination to advise a client to hold equities if they have a long-term planning horizon, or to hold bonds if they have a short-term planning horizon.

Although the expected value of a sequence of coin flips is zero, the variance in wealth is positive—increasing with the number of flips in the sequence. Under this model, the investor, having only a finite amount of capital, eventually will go bankrupt with 100-percent certainty given a sufficiently long horizon. A BF practitioner considers the limits of the CLT and concludes that variance in wealth levels is unacceptable for investors who must fund critical financial objectives; a MPT practitioner considers the limits of the CLT and concludes that the

risk-return tradeoff and its attendant opportunity cost evaluation should be quantified and closely monitored.

This brings the advisor to face another decision as shown in the BF cell of table 1, row 2:

- Do you model at-risk only asset allocations or do you include insured (floored) solutions? e.g., conservative vs. dissipative distribution policy using annuities and immunized bond positions

The Actuarial Approach

The actuarial approach involves the concept of “feasibility.” If the present value of liabilities is greater than the financial assets available to fund them, the client’s financial objectives are, at the time of measurement, technically infeasible. This is the concept of risk capacity.

Mapping, albeit crudely, Meder and Staub (2007) insights to private wealth management issues, we note: The feasibility of retirement objectives (current market value of financial assets – current cost of an annuity providing targeted lifetime income) is akin to the concept of funding ratio for corporate retirement plans. However, a negative corporate funding ratio implies that the plan has an in-the-money call on corporate assets should it become unable to fund promised future benefits; a negative funding ratio for a retired investor means that he or she can only hope that the situation changes—the investor owns no call options on outside financial resources.

The safety vs. opportunity cost argument is akin to the short-term hedge vs. long-term benefit protection argument. To restate this argument as it appears in

Meder and Staub (2007), from a short-term perspective, a plan sponsor defines risk as the possibility that the mimicking portfolio will fail to hedge liability price changes over the forthcoming period. A short-term hedging perspective suggests that the ideal asset portfolio consists of duration-matched bonds so that the change in values are offsetting. However, from a long-term perspective, the variability in the real dollar value of benefit payments is also a vital concern. Benefit costs are likely to increase and, therefore, the asset portfolio should account for inflation and growth factors in compensation—which, in turn, drive growth in dollar benefits. A good portfolio design model must deal with both planning horizons.

Asset management strategies that seem appropriate in the short term may undercut a portfolio’s chances of succeeding in the long term. An asset management approach that fails to consider and evaluate intelligently the claims of both the short and long term is difficult to defend.

In terms of avoiding depletion of principal:

- allocations tilted toward fixed income instruments may run an unacceptably high risk of depletion during the investor’s lifespan because realized return falls below required return;
- allocations tilted toward stocks may run an unacceptably high risk of depletion because realized downside volatility may exceed the portfolio’s drawdown capacity.

In terms of consumption variance:

- allocations tilted toward fixed income may erode purchasing power;
- allocations tilted toward stocks may necessitate substantial reductions in the investor’s standard of living if the sequence of realized returns is unfavorable.

The actuarial benchmarking issue—does an annuity cost benchmark constitute a reasonable way to contrast and compare differing retirement planning strategies—is emerging as an important topic in retirement income

planning. Shortfall-risk metrics and risk-of-ruin metrics remain important tools for assessing asset management strategies. Inclusion of an actuarial measurement of retirement feasibility triggers a new debate.

Commentators are split on whether to:

- annuitize as soon as possible lest a forthcoming bear market jeopardize the ability to secure threshold income, or
- delay annuitization in order to capture the expected equity risk premium and, potentially, enter into a lower-cost annuity contract issued at an older age.

The choice is between annuitizing now and resolving ambiguities surrounding the sources and amount of future income or waiting to annuitize as long as a delay remains a prudent and suitable investment management election.

Babbel and Merrill (2007) present a good example of the annuitize as soon as possible strategy. The authors suggest that a utility-maximizing investor will not pursue a strategy that leaves a positive probability of failing to support a threshold level of lifetime consumption. Their model directs the investor to allocate risk-free assets sufficient to support the minimum periodic income goal.

If the minimum consumption target requires periodic income greater than that available through government or corporate pension benefits, Babbel and Merrill (2007) advise the investor to annuitize immediately a portion of current wealth sufficient to fund the deficit. Excess wealth remains invested in a financial asset portfolio. Babbel and Merrill recommend that the investor allocate risk-free assets sufficient to support the minimum standard-of-living goal. In a multi-period context, the risk-free asset is an inflation-adjusted annuity. The authors argue for a bottom-up asset management approach, in which the investor, with little or no delay, converts financial assets into an annuity designed to provide threshold income. Only surplus wealth is allocated to a risky-asset portfolio

—a buy-an-annuity-and-invest-the-difference strategy.

If the amount of wealth allocated to the annuity is large, however, the investor may not have remaining funds sufficient to implement the optimal allocation to the risky-asset portfolio. Assuming that the risky-asset portfolio has a higher expected return than the annuity portfolio, the decrease in aggregate expected return (disutility) must be balanced against reduced uncertainty in future consumption (utility). The authors point out that the feedback loop plus the wealth constraint make an analytic solution impossible. This is a variation on the Meder and Staub (2017) asset management issue that DB plan sponsors must evaluate. It is also interesting to note how rational-school commentators develop recommendations that, in many respects, mirror those of behavioral finance practitioners.

In contrast to Babbel and Merrill, Fullmer (2007) espouses a top-down approach, where the option to annuitize is a last-resort asset management option. Fullmer asserts that the best strategy for managing retirement income risk is to annuitize when necessary—but not before.

The key to implementing a prudent portfolio management strategy is to evaluate continuously the option to annuitize financial assets. By exercising the option only when it is necessary to ensure a threshold standard of living, the investor takes full advantage of the time value of the annuitization option.

The key for leveraging this optionality is setting the projected cost to annuitize the investor's desired lifetime income stream as a wealth goal in the objective function. Doing so effectively transforms longevity risk into investment risk, because now it is the portfolio's job to preserve the ability to annuitize the desired lifetime income stream. ... By monitoring the investor's wealth relative to the current cost of annuitization, the decision to invest or annuitize can be continually evaluated by a financial adviser (Fullmer 2007).

This logic leads directly to a recommendation for a dynamic allocation strategy. Fullmer asserts, “When substantial cash flow risk is present, the objective function begins to take on more of the characteristics of a cash flow matching model.” The risk management approach mirrors the hurdle race problem in which the “provision” must exceed the cost of securing the threshold living standard through annuitization. The author terms this an “annuitization hurdle.” Under this risk management approach, the investor monitors the cost of buying an annuity to fund threshold income and compares this cost with the market value of assets remaining in the portfolio. The decision becomes how much of the portfolio surplus to put at risk before exercising the option to annuitize. The argument is that there is time value, if economically feasible, in the option to delay annuitization (where annuitization may be thought of as a type of actuarial flooring) until future events resolve an investor's uncertainties. Collins et al. (2015a) provides a more complete analysis of this topic.

As we move through the curve to the triangle to the rectangle perspectives at the level of model/system considerations, the debate shifts from a portfolio focus to a client focus. In the triangle perspective, a client diagnostic is achieved by differentiating the client's risk aversion in the domain of losses vs. the domain of gains. In the rectangle perspective, the client diagnostic is achieved by placing the discussion in the context of the Household Balance SheetSM, where the advisor faces the choice shown in the final cell of table 1, row 2:

- Do you model (and monitor) risk capacity before shortfall probabilities? e.g., feasibility before sustainability

In the rectangle perspective, understanding client circumstances and financial wherewithal is paramount. Advisors achieve such understanding by placing the discussion in the context of the Household Balance Sheet for private wealth investors; and, for trusts, by adhering to the provisions of Section 2(a) of the Uniform Prudent Investor Act as adopted and modified by state statute:

A trustee shall invest and manage trust assets as a prudent investor would, by considering the purposes, terms, distribution requirements, and other circumstances of the trust. In satisfying this standard, the trustee shall exercise reasonable care, skill, and caution.

Many retirees are current beneficiaries of irrevocable trusts, and the income from such trusts is often of great importance to maintaining an acceptable standard of living. Although coordination of trust and personal investor resources lies well outside the scope of this discussion, advisors should be aware of trust provisions—especially those that direct distributions in support of a beneficiary’s health, education, maintenance, and support (the HEMS ascertainable standards provisions).³

Although retired investors endeavor to implement investment strategies that offer an expectation for long-term sustainability, the issue of feasibility also arises. Does the current value of financial resources equal or exceed the current value of projected cash-flows and other critical liabilities? This issue is not new. You can find evidence in Yaari (1965), which seeks to determine the “optimal feasible consumption.”

We begin with Leibowitz and Henriksson (1989) and their research concerning the impact of incorporating shortfall constraints within the context of MPT analysis. MPT-based approaches were on the road to acknowledging critical economic objectives and designing shortfall-constrained optimal portfolios. Curve-based rational portfolio optimization was moving closer to triangle-based approaches.

About 15 years later, Nevins (2004), flatly states: “... the notion of an overall risk tolerance for each investor is flawed. Behavioral theorists have shown that investors have not just one but multiple attitudes about risk.”

Two years later, Brunel (2006) turns this observation into a proposed solution that matches goals to assets. The view is still investment centric, but personal goals and resources also form a basis for communications with the clients and for portfolio design by the analyst.

Behavioral portfolio theory (BPT) offers a rich set of empirical observations, cognitive hypotheses, psychological observations, and financial theory; and it should come as no surprise that research leads to differing, and often contradictory, investment prescriptions. See, for example, the brief discussion about whether clients are risk averse or risk prone in the domains of gains and losses (Meyer 2014; Blake et al. 2013).

Bordley and LiCalzi (2000) offer insight into how MPT and BPT might comfortably co-exist within the science of decision analysis. Their research considers the curve perspective—based on investor utility—as well as the triangle perspective based on goals, targets, and thresholds.

Bordley and LiCalzi (2000) offer this key conclusion: “... the problem of maximizing a multi-attribute utility function can be mapped to the problem of minimizing the probability of failure in a fault-tree (more generally, an event tree).” The cumulative distribution function of the two proposed probability densities for measuring client utility assumes an “S” shape as the client moves from the target/threshold value.

This argues that risk tolerance does not have a single value over the domain of gains vs. the domain of losses. In the field of decision analysis, one finds studies supporting a rational school, a behaviorist school, or a third alternative as a synthesis school.

Risk Capacity

Risk capacity is a measured accounting value, expressed in dollars. It derives from the Household Balance SheetSM (HHBSSM) analysis and is the difference between the net present value (and market value) of assets and the net present value of liabilities. A retiree’s risk capacity is a matter of judgment subject to the completeness (all relevant assets and liabilities) and accuracy (use of the proper inflation and discount rates) of the HHBS analysis. To understand how risk exposures connect with risk capacity, note that a strong HHBS evidences more risk capacity because there is a greater financial cushion to absorb risk. Some, or all, of this cushion can be exposed to risky assets and therefore invested as upside.

We find preliminary articulations of “risk capacity” in Nevins (2004) and others. Understanding risk capacity in this context leads to the definition of safety-first vs. probability-based asset management.

A series of studies articulates some of these concepts; see, for example, Brunel (2006) where one can see the distinction between curve-perspective for the design of portfolios and enhanced triangle-perspective for the communication of such portfolio designs to the client. The curves-become-triangles-become-rectangles process is clearer in Wilcox (2009) as he introduces

Table 1, Row 3: Output/Response Row			
Examples of Debates (and Contradictory Prescriptions)	Are You a Curve? Modern Portfolio Theory (MPT)	Are You a Triangle? Behavioral Finance (BF)	Are You a Rectangle? The Household Balance Sheet SM (HHBS SM)
Output [RESPONSE]	<p>What is the role of equity-weightings in asset allocation recommendations?</p> <p>e.g., buy-hold, constant-mix, constant proportion, declining or rising equity weightings</p>	<p>Do you recommend behavioral allocations?</p> <p>e.g., implementation as a single portfolio vs. implementation as multiple portfolios (e.g., goal-based asset allocation)</p>	<p>Do you recommend procedural allocations?</p> <p>e.g., mapping of risk exposures, risk management techniques allocations before asset allocations</p>

us to his discretionary wealth approach. A noteworthy, contemporary paper to Wilcox (2009) is Amenc et al. (2009).

The idea that retirement planning is fundamentally about funding liabilities (in contrast to optimizing assets) can be seen in Berkelaar and Kouwenberg (2010). Variations on these approaches range from brute force solutions such as Vertes (2013) to algorithmic solutions such as Mladina (2016).

Finally, the rank ordering of “feasibility” before “sustainability” before “investment performance” is an on-going discussion that we look forward to documenting in other papers. For now, we refer the reader to Ciccotello (2016).

For an extensive treatment of the importance of and distinctions between feasibility and sustainability in the context of the monitoring of retirement portfolios see Collins et al. (2015b).

Table 1, Row 3: Notes about Output/Response Row

The advisor’s selections of inputs and models lead to output choices shown in the output/response row (table 1, row 3).

Differences between rationalist/behavioralist views concerning the role of equity within a retirement income portfolio marks, according to some commentators, a distinctive boundary in portfolio construction and asset management philosophies. Although more recent published studies represent a convergence of viewpoints, early research offers starkly different prescriptive recommendations.

Ho et al. (1994) test the financial planning advice that retirees should own mostly bonds to produce retirement income. They conclude that the greatest retirement risk is outliving capital and that 100-percent equity portfolios are essential for most retirees unless the investor has attained advanced age or spends at a high-value wealth/consumption ratio rate.

Bengen (1994), building on an historical back-testing methodology popularized by

Bierwirth (1994), examined the correspondence between equity weighting and retirement spending sustainability. Bengen (1994) examines equity allocations of 0, 25, 50, 75, and 100 percent paired with withdrawal rates (percentage of initial portfolio value maintained over the planning horizon on a constant dollar basis) of 1, 2, 3, 4, 5, 6, 7, and 8 percent. The variable of interest is the minimum number of years that a portfolio will last given starting dates of 1926 through 1976. For example, at a 1-percent withdrawal rate, all portfolio allocations last for 50 years. A 2-percent withdrawal rate cannot be sustained over the 50-year period with a 100-percent Treasury note allocation. A 3-percent withdrawal rate requires more than a 25-percent allocation to equities. Across all withdrawal rates, an allocation of 50-percent to 75-percent equities resulted in the highest survival percentages. These allocations also produced substantial ending portfolio wealth: “... I think it is appropriate to advise the client to accept a stock allocation as close to 75% as possible, and in no cases less than 50%” (Bengen 1994).

Cooley et al. (2001) represent a high-water mark in the use of historical, back-testing methodology. The data indicate that long-term sustainability at nominal withdrawal rates higher than 6–7 percent requires at least a 50-percent equity weighting for payout periods longer than 25 years. High success rates for inflation-adjusted withdrawals require limiting the withdrawal rate to 5 percent or less while maintaining an equity weighting of at least 50 percent. The authors note: “the lower withdrawal rates of 3% and 4% recommended by some analysts appear to be excessively conservative for portfolios with at least 50% stock, unless the investor wishes to leave a substantial portion of the initial retirement portfolio to his/her heirs.”

Hughen et al. (2002) extends the Cooley et al. (2001) research to portfolios operating in a taxable environment. A portfolio with an initial value of \$1 million is invested under five asset allocations: 100-percent, 75-percent, 50-percent, 25-percent, and 0-percent equity. Calculations are

expressed in both nominal dollar terms and in constant dollars. Withdrawals occur annually (at the time of portfolio rebalancing) and the withdrawal amount factor is 1 plus the ratio of the current year’s consumer price index (CPI) number to the previous year’s number. The total dataset encompasses 55 overlapping 20-year periods, 50 overlapping 25-year periods, and 45 overlapping 30-year periods. For 20-year periods, there is a high success rate for all allocation/withdrawal combinations provided that the withdrawal rate is 5 percent or less. For withdrawals at a higher rate, the 0-percent equity allocation exhibits dramatically increasing failure rates. The 100-percent equity allocation exhibits the lowest failure rates for high withdrawals—6 percent to 12 percent: “For all three retirement periods, the 100% equity allocation has the lowest percentage of failures at withdrawals above 7%.”

Not all studies recommended a strong tilt towards stocks. Jarrett and Stringfellow (2000) recommends loading for intermediate government bonds. The authors test four spending policies (constant nominal, increase by set factor, increase by inflation factor, and withdraw a fixed percentage of market value) across three portfolio objectives: (1) final portfolio = zero, (2) nominal principal is preserved, and (3) inflation-adjusted principal is preserved. All tests are historical back-tests over rolling periods of various length. Portfolios are rebalanced annually. Sample period is 1926 through 1998. They conclude: “... a portfolio of 100% large cap stocks would allow you to withdraw 3.92% a year over a 20-year time frame ... if the fund contained 25% large cap and 75% intermediate government bonds, the withdrawals can increase to 6.7% ...”

By 2005, serious studies examine more dynamic approaches to spending, allocation, and downside risk probabilities. Dus et al. (2005) is a good example. Many investors choose to self-annuitize under a phased withdrawal approach. However, a fixed amount withdrawal election carries the risk of outliving financial resources. A fraction of remaining wealth withdrawal strategy avoids this risk, but the periodic amount

withdrawn may be substantially higher or lower than the fixed benefit amount. (Note: The retirement income problem involves a tradeoff between budgetary certainty with an attendant risk of ruin and budgetary uncertainty with an attendant risk of insufficient periodic income.)

Dus et al. (2005) seek to compare alternative retirement income strategies. Traditionally, financial economists approach the problem by determining the plan that maximizes discounted expected utility of uncertain future consumption and bequests. Most models based on utility functions assume time separable utility and constant relative risk aversion. These limiting assumptions however may not represent explicit measures of a retiree's risk preferences. Therefore, the authors elect a risk-value model where reward is defined as expected return from any retirement income strategy and risk is defined as the possibility of not reaching the desired level of consumption ("probability of consumption shortfall").

The authors stress that valid risk metrics must account for both the timing and the magnitude of losses. They anticipate the argument made by Kitces (2012a). The article demonstrates the benefit of payout modeling as opposed to investment risk modeling—income patterns are graphed over time instead of dollar wealth patterns. Some income patterns provide:

- back-loaded retirement benefits
- some provide stable benefits
- some exhibit benefits that decline over time

Depending on the nature of the investor's utility of consumption function, one pattern may be preferred over another despite the fact that it produces a lower overall present value.

Several articles question the validity of research methods based on pure empiri-

cism. Mathematical necessity is not found in historical outcomes; and reliance on observed data series only provides the investigator with a single outcome sample. Albrecht et al. (2001) is a good example of this line of thought. Empirical evidence may suggest that stocks always beat bonds in the long run and, therefore, are the true riskless investment. According to this argument, in the long run, bonds are a redundant asset class. Advocates of this position usually take a purely historical approach. However, when the approach utilizes 10-, 15- or 20-year overlapping periods, the resulting returns have a high degree of correlation. "This results in a serious estimation bias. A high degree of statistical significance requires independent returns based on non-overlapping ... periods" (Albrecht et al. 2001).

“The bottom line is that choosing between immediate annuities and safe withdrawal rates is not a decision about whether to use a floor-with-upside approach; it’s about choosing which floor is preferable in light of the tradeoffs the decision entails.”

Bodie (2002) contributes to the load for/load against equity debate. He demonstrates that consumption demands create path dependencies; the average return is not a valid predictor of terminal portfolio wealth because low early returns may exhaust wealth despite higher-than-average late returns. In the long run, according to Bodie, the only safe investments are inflation-protected bonds and real annuities.

Divergence in financial advice became the norm rather than the exception. Path dependency, sequence risk, and equity volatility prompted many commentators either to recommend fixed income portfolios; or, at least to recommend starting retirement in a conservative posture. The approach justifies a safety-first portfolio preference criterion for asset management. Portfolios that are initially conservative might be better suited to meet interim

goals (wealth targets and consumption) at a higher probability. If future surplus develops, then the investment strategy can become less conservative.

Other commentators recommend the opposite; and a growing number of advisors explore adaptive spending and asset allocation policies. A good place to end this brief discussion of the use of equity vs. the use of fixed income flooring is Kitces (2012b).

An annuity strategy provides a floor on income that cannot be outlived. However, according to Kitces (2012b), the 4-percent withdrawal rule is also effectively a floor strategy. Although it does not guarantee lifetime income, it has never failed in market history. Thus, an annuity may be seen as an alternative way to achieve safe withdrawals but with a loss of liquidity as well as upside return potential. The safe 4-percent withdrawal rate is calculated based on a 30-year planning horizon whereas the annuity—which is backed by an insurance company guarantee,

pays periodic income irrespective of the annuitant's actual lifespan. "To truly fail, the couple needs to be unlucky enough to live through an investment environment worse than any found in history (i.e., no principal left at the end) and be the (approximately) one couple in six who are still alive at the 30-year time horizon. When you combine low-probability investment disasters and low-probability longevity scenarios, you end up with some astonishingly low-probability scenarios, many of which could be further 'saved' by small midcourse corrections."

Kitces (2012b) argues that "... extraordinary investment shocks that could destroy a 30-year safe withdrawal rate could also threaten an insurance company... the failure rate of a 4% withdrawal rate is about the same as the failure rate of an insurance company rated AA or better... Simply put, the tail risks are correlated."

“The bottom line is that choosing between immediate annuities and safe withdrawal rates is not a decision about whether to use a floor-with-upside approach; it’s about choosing which floor is preferable in light of the tradeoffs the decision entails.” This implies that a 4-percent withdrawal strategy (“... has never failed in market history”) is almost as conservative a strategy for producing safe and sustainable income as is the purchase of an annuity. This is especially true when the probability of insurance company default is considered.

Both a 4-percent withdrawal rate strategy and an annuity offer floor protection, but the annuity truncates upside potential.

Output/Response Debate around Rectangle Portion

Evensky (2006) proposed an alternative to flooring using custom total return portfolios as the default client recommendation. The perceptive reader may notice that the total-return vs. safety-first debate is a variation on the utility/decision/investor choice controversy outlined earlier in this paper: economic state-of-nature (probability) vs. payoff (financial consequences to investor). Pursuing a flooring-followed-by-risky-asset-(pyramid) approach resolves certain aspects of ambiguity (i.e., uncertainty regarding future investment returns) at the cost of a budget constraint (loss of opportunity to earn the expected equity risk premium) that compromises the investor’s ability to occupy a position on the efficient frontier tangent to his or her utility—investor indifference isoquant—curve.

From a practitioner perspective, there is a “forever-war” aspect to these debates. This confirms the wisdom of developing round descriptions of the clients in order to reach prudent, defensible, and administratively practical recommendations. The scope of inquiry includes the following questions:

Retirement feasibility: Is the NPV of assets greater than the NPV of liabilities?

Retirement sustainability: What is the shortfall risk of a curve, triangle, or rectangle strategy to critical standard-of-living goals?

Retirement performance: As time unfolds, does the strategic allocation meet or exceed the threshold return necessary for the continued success of the retirement plan?

Translating this into the language of fiduciary standards of practice for trustees, suggests that a test for an objective’s feasibility is an aspect of *care*; asset management for income sustainability is an aspect of *skill*; and, monitoring retirement performance is an aspect of *caution*. But, care, skill, and caution are the defining characteristics of prudent investment management and trust administration: “A trustee shall invest and manage trust assets as a prudent investor would, by considering the purposes, terms, distribution requirements, and other circumstances of the trust. In satisfying this standard, the trustee shall exercise reasonable care, skill, and caution.” For additional commentary see Collins and Stampfli (2001) and Collins (2007, 2011).

The curve, triangle, rectangle research project further clarifies and contributes to this discussion. Starting with Brunel (2003), we see a recommendation to pay close attention to four client investment goals (liquidity, income, capital preservation, and growth). The recommendation is helpful; but Brunel remains focused primarily on financial assets instead of client circumstances. One result of such a focus is the failure to consider key risks such as client longevity.

With Ziemba (2003), a similar pattern repeats: “The optimum occurs for investors holding four funds—the market portfolio, the hedge portfolio for the state variable, the hedge portfolio for liabilities, and the riskless asset.”

Chhabra (2005) goes further and asserts: “A major conclusion of this work is that, for the individual investor, Risk Allocation should precede Asset Allocation.”

Leibowitz (1989), Fowler (2006), and the “Great Debate” that includes Brunel and Chhabra—and continues to this day—mirrors the articulation of the safety-first vs. probability-based debate in 2012.

More recently, the debate has moved to exploring both product-based and systems-based approaches for enterprise-level implementations that, in some cases, reconcile the curve and triangle perspectives. See, for example, Das et al. (2010), Merton (2014), Sironi (2016), and Martellini (2016).

These debates illustrate the genealogy of the financial industry’s process of adopting a progressively more integrative view of both the portfolio (curve) and the client (triangle) into a more integrated framework (rectangle) of feasibility, sustainability, and performance that can be communicated easily to the client. ●

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Endnotes

1. In mathematics, a Taylor series is a representation of a function as an infinite sum of terms that are calculated from the values of the function’s derivatives at a single point. https://en.wikipedia.org/wiki/Taylor_series.
2. Fisher utility reflects a strong preference for current consumption over future consumption. In a retirement planning context, an investor with Fisher utility wishes to front-load retirement income. The investor is willing to incur the risk of reduced spending ability should he or she live to an advanced age.
3. The HEMS ascertainable standard permits a trustee to make distributions of trust-owned assets in support of a beneficiary’s health, education, and maintenance/support needs. Ordinarily, a trustee may not make discretionary distributions for other needs (e.g., extraordinary gifts) unless such distributions are permitted by the governing trust document.

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